

Some Siegel threefolds with a Calabi-Yau model

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Abstract. We describe some examples of projective Calabi-Yau manifolds which arise as desingularizations of Siegel threefolds. There is a certain explicit product of six theta constants which defines a cusp form of weight three for a certain subgroup of index two of the Hecke group $\Gamma_{2,0}[2]$. This form defines an invariant differential form for this group and for any subgroup of it. We study the question whether the Satake compactification for such a subgroup admits a projective desingularization on which this differential form is holomorphic and without zeros. Then this desingularization is a Calabi-Yau manifold. We shall prove: For any group between $\Gamma_2[2]$ and $\Gamma_{2,0}[2]$ there exists a subgroup of index two which produces a (projective) Calabi-Yau manifold. The proof rests on a detailed study of this cusp form and on Igusa's explicit desingularization of the Siegel threefolds with respect to the principal congruence subgroup of level $q > 2$ (we need $q = 4$). For a particular case we produce the equations for the corresponding Siegel threefold.

Mathematics Subject Classification (2010): 11F46 (primary); 14J32 (secondary).