Deformations and automorphisms: a framework for globalizing local tangent and obstruction spaces

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Abstract. Building on Schlessinger's work, we define a framework for studying geometric deformation problems which allows us to systematize the relationship between the local and global tangent and obstruction spaces of a deformation problem. Starting from Schlessinger's functors of Artin rings, we proceed in two steps: we replace functors to sets by categories fibered in groupoids, allowing us to keep track of automorphisms, and we work with deformation problems naturally associated to a scheme X, and which naturally localize on X, so that we can formalize the local behavior. The first step is already carried out by Rim in the context of his homogeneous groupoids, but we develop the theory substantially further. In this setting, many statements known for a range of specific deformation problems can be proved in full generality, under very general stack-like hypotheses.