

Kähler manifolds and their relatives

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Abstract. Let M_1 and M_2 be two Kähler manifolds. We call M_1 and M_2 *relatives* if they share a non-trivial Kähler submanifold S , namely, if there exist two holomorphic and isometric immersions (Kähler immersions) $h_1 : S \rightarrow M_1$ and $h_2 : S \rightarrow M_2$. Moreover, two Kähler manifolds M_1 and M_2 are said to be *weakly relatives* if there exist two locally isometric (not necessarily holomorphic) Kähler manifolds S_1 and S_2 which admit two Kähler immersions into M_1 and M_2 respectively. The notions introduced are not equivalent (*cf.* Example 2.3). Our main results in this paper are Theorem 1.2 and Theorem 1.4. In the first theorem we show that a complex bounded domain $D \subset \mathbb{C}^n$ with its Bergman metric and a projective Kähler manifold (*i.e.* a projective manifold endowed with the restriction of the Fubini–Study metric) are not relatives. In the second theorem we prove that a Hermitian symmetric space of noncompact type and a projective Kähler manifold are not weakly relatives. Notice that the proof of the second result does not follow trivially from the first one. We also remark that the above results are of local nature, *i.e.* no assumptions are used about the compactness or completeness of the manifolds involved.

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