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Energy improvement for energy minimizing functions in the complement of generalized Reifenberg-flat sets

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Abstract. Let P be a hyperplane in $\mathbb{R}^N$, and denote by d_H the Hausdorff distance. We show that for all positive radius $r < 1$ there is an $\varepsilon > 0$, such that if K is a Reifenberg-flat set in $B(0,1) \subset \mathbb{R}^N$ that contains the origin, with $d_H(K, P) \leq \varepsilon$, and if u is an energy minimizing function in $B(0, 1)\backslash K$ with restricted values on $\partial B(0, 1)\backslash K$, then the normalized energy of u in $B(0, r)\backslash K$ is bounded by the normalized energy of u in $B(0, 1)\backslash K$. We also prove the same result in $\mathbb{R}^3$ when K is an ε-minimal set, that is a generalization of Reifenberg-flat sets with minimal cones of type $\mathbb{Y}$ and $\mathbb{T}$. Moreover, the result is still true for a further generalization of sets called $(\varepsilon, \varepsilon_0)$-minimal. This article is a preliminary study for a forthcoming paper where a regularity result for the singular set of the Mumford-Shah functional close to minimal cones in $\mathbb{R}^3$ is proved by the same author.

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