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## Combinatorial realization of the Thom-Smale complex via discrete Morse theory

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**Abstract.** In the case of smooth manifolds, we use Forman's discrete Morse theory to realize combinatorially any Thom-Smale complex coming from a smooth Morse function by a pair triangulation-discrete Morse function. As an application, we prove that any class of homologous vector fields on a smooth oriented closed 3-manifold can be realized by a perfect matching on the Hasse diagram of a triangulation of the manifold.

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