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On normal and non-normal holomorphic functions on complex Banach manifolds

PETER V. DOVBUSH

Abstract. Let X be a complex Banach manifold. A holomorphic function $f : X \to \mathbb{C}$ is called a normal function if the family $\mathcal{F}_f = \{f \circ \varphi : \varphi \in \mathcal{O}(\Delta, X)\}$ forms a normal family in the sense of Montel (here $\mathcal{O}(\Delta, X)$ denotes the set of all holomorphic maps from the complex unit disc into X). Characterizations of normal functions are presented. A sufficient condition for the sum of a normal function and non-normal function to be non-normal is given. Criteria for a holomorphic function to be non-normal are obtained.

These results are used to draw one interesting conclusion on the boundary behavior of normal holomorphic functions in a convex bounded domain D in a complex Banach space V. Let $\{x_n\}$ be a sequence of points in D which tends to a boundary point $\xi \in \partial D$ such that $\lim_{n\to\infty} f(x_n) = L$ for some $L \in \overline{\mathbb{C}}$. Sufficient conditions on a sequence $\{x_n\}$ of points in D and a normal holomorphic function f are given for f to have the admissible limit value L, thus extending the result obtained by Bagemihl and Seidel.

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