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A priori estimates for weak solutions of complex Monge-Ampère equations

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Abstract. Let X be a compact Kähler manifold and ω be a smooth closed form of bidegree $(1, 1)$ which is nonnegative and big. We study the classes $\mathcal{E}_\chi(X, \omega)$ of ω-plurisubharmonic functions of finite weighted Monge-Ampère energy. When the weight χ has fast growth at infinity, the corresponding functions are close to be bounded.

We show that if a positive Radon measure is suitably dominated by the Monge-Ampère capacity, then it belongs to the range of the Monge-Ampère operator on some class $\mathcal{E}_\chi(X, \omega)$. This is done by establishing a priori estimates on the capacity of sublevel sets of the solutions.

Our result extends those of U. Cegrell's and S. Kolodziej's and puts them into a unifying frame. It also gives a simple proof of S. T. Yau's celebrated a priori $\mathcal{C}^0$-estimate.

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