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# Sharp upper bounds for a singular perturbation problem related to micromagnetics

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**Abstract.** We construct an upper bound for the following family of functionals $\{E_\varepsilon\}_{\varepsilon>0}$, which arises in the study of micromagnetics:

$$E_\varepsilon(u) = \int_\Omega \varepsilon|\nabla u|^2 + \frac{1}{\varepsilon}\int_{\mathbb{R}^2} |H_u|^2.$$

Here $\Omega$ is a bounded domain in $\mathbb{R}^2$, $u \in H^1(\Omega, S^1)$ (corresponding to the magnetization) and $H_u$, the demagnetizing field created by $u$, is given by

$$\begin{cases} \operatorname{div}(\tilde{u} + H_u) = 0 & \text{in } \mathbb{R}^2, \\ \operatorname{curl} H_u = 0 & \text{in } \mathbb{R}^2, \end{cases}$$

where $\tilde{u}$ is the extension of $u$ by 0 in $\mathbb{R}^2 \setminus \Omega$. Our upper bound coincides with the lower bound obtained by Rivière and Serfaty.

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